CAP in QPA

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First CAP Days, 2018-08-28

(Parts of this presentation are adapted from *Introduction to QPA*,
my joint talk with Øyvind Solberg at the Third GAP Days in Trondheim, 2015)

What is QPA?

- QPA: “Quivers and Path Algebras”
- GAP package for computations with quotients of path algebras and their modules

Part 1

So what are these “quivers”, anyway?

Quivers

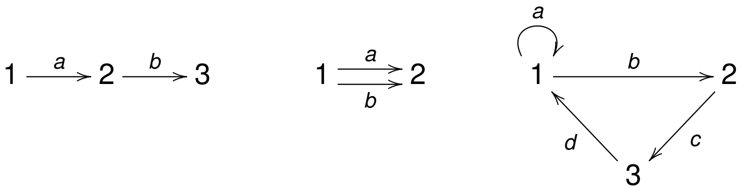
Quivers

$$\begin{tikzcd} 1 \arrow[r, "a"] & 2 \arrow[r, "b"] & 3 \end{tikzcd}$$

$$\begin{tikzcd} 1 \arrow[r, shift left, "a"] \arrow[r, shift right, "b"'] & 2 \end{tikzcd}$$

$$\begin{tikzcd} 1 \arrow[loop above, "a"] \arrow[rr, "b"] & & 2 \arrow[dl, "c"] \\ & 3 \arrow[ul, "d"] & \end{tikzcd}$$

Quivers



Quiver: oriented graph (loops and multiple edges allowed)

Paths

$$Q\colon\ 1 \xrightarrow{a} 2 \xrightarrow{b} 3$$

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- Length 1: a, b (arrows)

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Paths in Q:

- Length 0: e_1, e_2, e_3 (vertices/trivial paths)
- Length 1: a, b (arrows)
- Length 2: ba (concatenation of a and b)

Path algebras

$$\begin{tikzcd} Q\colon & 1 \arrow[r, "a"] & 2 \arrow[r, "b"] & 3 \end{tikzcd}$$

k a field

Path algebras

$$\left.\begin{array}{l} Q\colon\ 1 \xrightarrow{a} 2 \xrightarrow{b} 3 \\ k \text{ a field} \end{array}\right\} \rightsquigarrow \text{path algebra } kQ$$

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- Basis: $\{e_1, e_2, e_3, a, b, ba\}$
- Multiplication:

$$\begin{aligned} e_1 \cdot e_1 &= e_1 \\ e_1 \cdot e_2 &= 0 \\ e_2 \cdot a &= a \\ e_1 \cdot a &= 0 \\ b \cdot a &= ba \end{aligned}$$

Path algebras

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$$e_1 \cdot a = 0$$

$$b \cdot a = ba$$

$\cdot$	e_1	e_2	e_3	a	b	ba
e_1	e_1	0	0	0	0	0
e_2	0	e_2	0	a	0	0
e_3	0	0	e_3	0	b	ba
a	a	0	0	0	0	0
b	0	b	0	ba	0	0
ba	ba	0	0	0	0	0

Representations

Given

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Want to make a representation R of Q.

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Start with the quiver, and put

— a vector space at each vertex,

Representations

Given

$$Q\colon\ 1 \xrightarrow{a} 2 \xrightarrow{b} 3$$

Want to make a representation R of Q.

$$R\colon\ V_1 \xrightarrow{f_a} V_2 \xrightarrow{f_b} V_3$$

Start with the quiver, and put

- — a vector space at each vertex,
- — a linear transformation on each arrow.

Homomorphisms of representations

$$\begin{tikzcd} R: & V_1 \arrow[r, "f_a"] & V_2 \arrow[r, "f_b"] & V_3 \\ S: & W_1 \arrow[r, "g_a"] & W_2 \arrow[r, "g_b"] & W_3 \end{tikzcd}$$

Homomorphisms of representations

$$\begin{tikzcd}
R\colon \arrow[d, "h"] & V_1 \arrow[r, "f_a"] & V_2 \arrow[r, "f_b"] & V_3 \\
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S\colon & W_1 \arrow[r, "g_a"] & W_2 \arrow[r, "g_b"] & W_3
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A homomorphism $h\colon R \to S$ is given by:

- linear maps h_i for every vertex i,
- commuting with the linear maps for the arrows.

Representations and modules

$$\operatorname{mod} kQ \simeq \operatorname{Rep}_k Q$$

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finitely generated *kQ*-modules

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finitely generated *kQ*-modules

representations of *Q* over *k*

Part 2

A brief history of QPA

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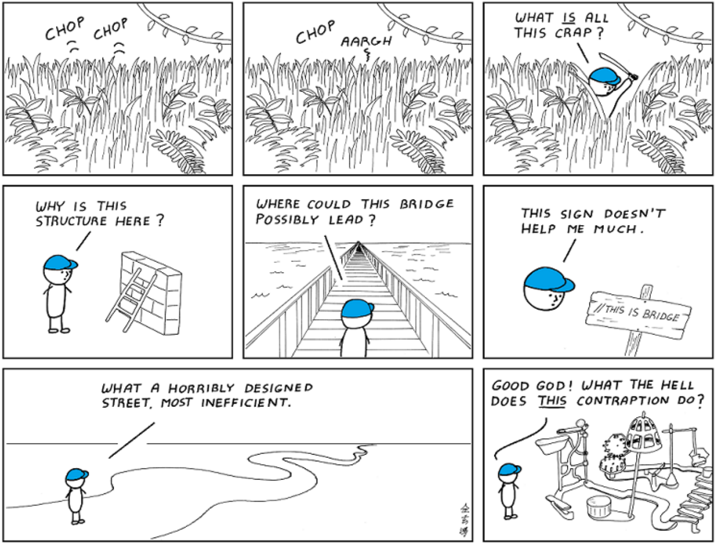
- Started by Ed Green at Virginia Tech ca 1998(?)
- Originally called HOPF
- Changed and extended by many people over the years
- Name changed to QPA at some point
- Adopted by Øyvind Solberg (NTNU, Trondheim) in 2010

QPA, ca. 2014



http://www.abstrusegoose.com/432 (CC BY-NC 3.0 US)

What do we do?

```
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What do we do?

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… and so on

QPA version 2

- Complete rewrite of QPA
- Aim: Cleaner and more consistent code
- Started in 2015, still not ready to replace QPA1

Part 3

CAP and us

Some of the problems in QPA1

- Much of QPA is really about certain abelian categories and functors
- But not explicitly: Nothing in GAP lets us say “this is a category” or “this is a functor”
- Have many functions that deal with categorical and functorial aspects, but not in a consistent or complete manner
- Have arbitrarily added (and named) functions that are “too general” – they are applicable in any (abelian) category

Example: Kernel

$$\begin{tikzcd}M \arrow[r, "f"] & N\end{tikzcd}$$

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$$\operatorname{Ker} f \qquad M \xrightarrow{f} N$$

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$$\begin{tikzcd}
\operatorname{Ker} f \arrow[r, "\text{inc}"] & M \arrow[r, "f"] & N
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X \arrow[u, dashed] \arrow[ur] & &
\end{tikzcd}$$

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M \arrow[r, "f"] \arrow[d, "\mu"] & N \arrow[d, "\nu"] \\
M' \arrow[r, "f'"] & N'
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Example: Chain complexes

$$C\colon\ \cdots \to C_2 \xrightarrow{d_2} C_1 \xrightarrow{d_1} C_0 \xrightarrow{d_0} C_{-1} \xrightarrow{d_{-1}} C_{-2} \to \cdots$$

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Represented by repeating list (r_1, r_2, r_3) of differentials:

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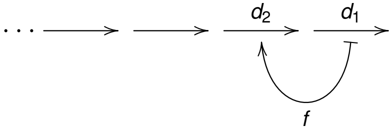

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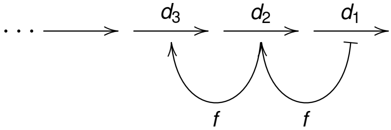

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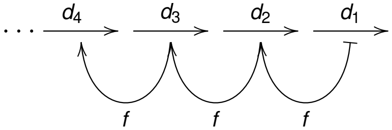

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… and then we learned about CAP.

Result

- QPA2 is written with CAP in mind (almost) from the beginning

Categories in QPA2

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vec *k*

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$$\begin{tikzcd}
\operatorname{Rep}_k Q \\
\operatorname{vec} k \arrow[u, squiggly]
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Functors in QPA2: Hom and tensor (work in progress)

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- Categories and functors are objects
- CAP makes our code better structured

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— *Heavy use of CAP could slow down the code*
. . . but there might be workarounds, such as turning off caching

— *Confusing terminology: “GAP categories” and “CAP categories”*

Possible disadvantages: Response

— *CAP is big and complex*
. . . but still reasonably easy to use in simple ways

— *Deeply nested function calls complicate debugging*
. . . but that is probably necessary

— *Heavy use of CAP could slow down the code*
. . . but there might be workarounds, such as turning off caching

— *Confusing terminology: "GAP categories" and "CAP categories"*
. . . but that is GAP's fault, not CAP's

Wish list

Wish list

- Parametrized categories

Wish list

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Wish list

- Parametrized categories: $\operatorname{mod} * \rightsquigarrow \operatorname{mod} A$

Wish list

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Wish list

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  Suggestion: A cat wearing a cap with the text “CAP”